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B_s Mesons using Staggered Light Quarks

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Last year we proposed using staggered fermions as the light quarks, combined with nonrelativistic heavy quarks, in simulations of heavy-light mesons. A first round of tests which focuses on the B_s meson has been completed using quenched lattices, and results are presented here for the kinetic B_s mass, the $B_s^* - B_s$ splitting, and f_{B_s}. The next project, already underway, is to compute the B and B_s decay constants and spectra on the $n_f = 2+1$ and 3 MILC lattices. We report on progress with one set of these configurations.

1. INTRODUCTION

Although a variety of lattice methods have been used to simulate the heavy quark within a heavy-light meson, the light quark description has not been studied as thoroughly; only Wilson-like fermions have been used in large scale simulations to date. Recent simulations of light hadrons suggest, however, that improved staggered fermions are superior to Wilson-like fermions, particularly for small quark masses. Improved staggered fermions have an exact chiral symmetry at zero mass, no $\mathcal{O}(a)$ errors and excellent scaling, and they require significantly less computational expense. These benefits will facilitate the study of leptonic decay constants and form factors in semileptonic decays as the physical light quark limit is approached.

We have been studying the coupling of staggered light quarks to nonrelativistic heavy quarks for simulation of heavy-light mesons. Since our initial study last year [1] we have resolved a question regarding the improved staggered action on a very coarse $8^3 \times 20$ lattice and have completed a calculation of the B_s mass, $B_s^* - B_s$ mass splitting, and f_{B_s} on a finer $12^3 \times 32$ lattice. We present those results in Section 2. In Section 3 we discuss progress with this simulation method on the $2+1$ flavor MILC configurations.

Given the constraints of this write-up we refer the reader to [1] for a reminder of our method for constructing operators out of NRQCD and naive fields and to [2,1] for a discussion of the constrained curve fitting we employ in extracting energies and matrix elements. A forthcoming paper [3] will give a detailed description of the methods used to obtain the results presented here.

2. QUENCHED RESULTS

The quenched configurations were generated using the tadpole-improved tree-level Symanzik action. Last year we reported difficulty in obtaining reasonable fits for heavy-light correlators on a lattice with $1/a = 0.8$ GeV when the $\mathcal{O}(a^2)$ staggered action (AsqTad) was used [1]. By the process of elimination, it was determined that the extended 3-link hopping of the Naik term in the temporal direction was responsible for the bad fits. This term is known to give rise to energetic negative-norm, ghost-like solutions to the free fermion dispersion relation. Since these states have energies proportional to $1/a_t$, their effects on correlators should become negligible as a_t decreases. Indeed on an anisotropic lattice with $1/a_t = 3.7$ GeV reasonable fits are obtained

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Table 1
Summary of fits to pseudoscalar heavy-light correlators.

volume action	$1/a_s$ (GeV)	a_s/a_t	$\chi^2_{\rm aug}$/DoF	$E_{\rm sim}(\mathbf{p}=0)$ (MeV)
$8^3\times 20$ 1-link	0.8	1.0	0.59	735(10)
$8^3\times 20$ AsqTad	0.8	1.0	8.93	–
$8^3\times 48$ AsqTad	0.7	5.3	1.59	790(36)
$8^3\times 48$ AsqTad w/o $\hat{t}$-Naik	0.7	5.3	1.03	901(19)
$12^3\times 32$ 1-link	1.0	1.0	0.48	873(9)
$12^3\times 32$ AsqTad	1.0	1.0	0.96	765(9)

with and without the temporal Naik term (see Table 1). Similarly, on an isotropic lattice with $1/a = 1.0$ GeV, there is no significant contamination from negative norm states; a fit is shown in Fig. 1.

We focus now on results on the $12^3\times 32$ lattice, where we used the AsqTad action with $am_0 = 0.10$ and an $\mathcal{O}(\Lambda_{\rm QCD}/M_0, a^2)$-improved NRQCD action with $aM_0 = 5.0$. The bare quark masses were estimated to coincide with the strange and bottom masses, respectively. Meson masses can be computed using either finite momentum correlators to construct the kinetic mass

$$M_{\rm kin} \;=\; \frac{|\,\mathbf{p}|^2 - [E_{\rm sim}(\mathbf{p}) - E_{\rm sim}(0)]^2}{2[E_{\rm sim}(\mathbf{p}) - E_{\rm sim}(0)]} \qquad (1)$$

or using perturbation theory

$$M_{\rm pert} \;=\; E_{\rm sim}(0) + Z_M M_0 - E_0 \qquad (2)$$

where Z_M is the heavy quark mass renormalization and E_0 is the self-energy constant. For this lattice and $aM_0 = 5.0$ we find $Z_M M_0 - E_0 = M_0 - 0.890\,\alpha_s + M_0\mathcal{O}(\alpha_s^2)$. The result is $M_{\rm pert} = 5.51 \pm 0.45$ GeV where the $\mathcal{O}(\alpha_s^2)$ uncertainty leads to the quoted error. The comparison of $M_{\rm kin}$ to $M_{\rm pert}$ is shown in Fig. 2.

Mass splittings can be computed directly from differences of simulation energies. We find $M_{B_s^*} - M_{B_s} = 25 \pm 5$ MeV, much smaller than the splitting expected from the experimental $M_{B^*} - M_B = 46$ MeV, but in agreement with other quenched results.

Matrix elements of the temporal QCD axial vector current A_0 are evaluated up through $\mathcal{O}(\Lambda_{\rm QCD}/M_0, \alpha_s, \alpha_s/aM_0)$ by computing matrix elements of

$$J_0^{(0)} \;=\; \overline{q}\,\gamma_5\gamma_0\,Q \qquad (3)$$

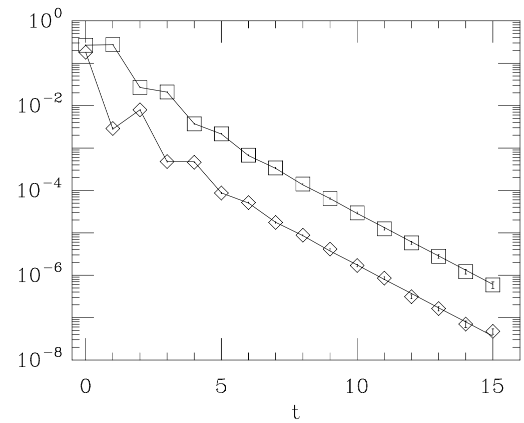

Figure 1. Squares show the $J_0^{(0)\dagger}(t)J_0^{(0)}(0)$ correlators on the $12^3\times 32$ lattice, and diamonds show the $J_0^{(0)\dagger}(t)J_0^{(1)}(0)$ correlator times -1.

$$J_0^{(1)} \;=\; -\frac{1}{2M_0}\overline{q}\,\gamma_5\gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q \qquad (4)$$

and using the operator matching relation

$$A_0 \;\doteq\; Z_{A_0} J_0^{(0)} + J_0^{(1,{\rm sub})} \qquad (5)$$

where the $\mathcal{O}(\Lambda_{\rm QCD}/M_0)$ contributions are absorbed into

$$J_0^{(1,{\rm sub})} \;\equiv\; J_0^{(1)} - \alpha_s\zeta_{10} J_0^{(0)}\,. \qquad (6)$$

With $aM_0 = 5.0$ we compute $Z_{A_0} = 1 + (0.208 \pm 0.003)\alpha_s$ and $\zeta_{10} = -0.0997$. Fits to appropriate local-local correlators yield

$$\frac{\langle 0|J_0^{(1,{\rm sub})}|B_s\rangle}{\langle 0|J_0^{(0)}|B_s\rangle} \;=\; -\,0.034 \pm 0.004\ ({\rm stat}). \qquad (7)$$

Applying (5) and

$$\langle 0|A_0|B_s\rangle \;\equiv\; f_{B_s} M_{B_s} \qquad (8)$$

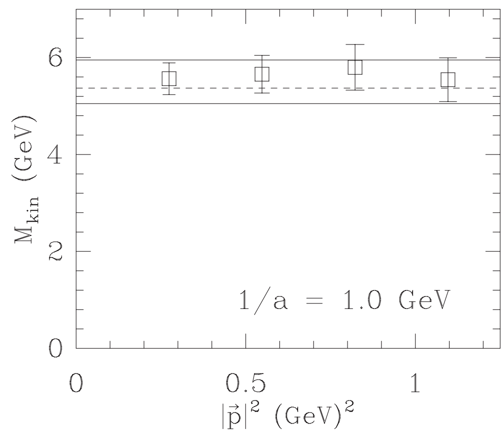

Figure 2. Kinetic B_s mass on the quenched lattice. The solid lines mark the range of $M_{\rm pert}$ and the dashed line marks the experimental mass.

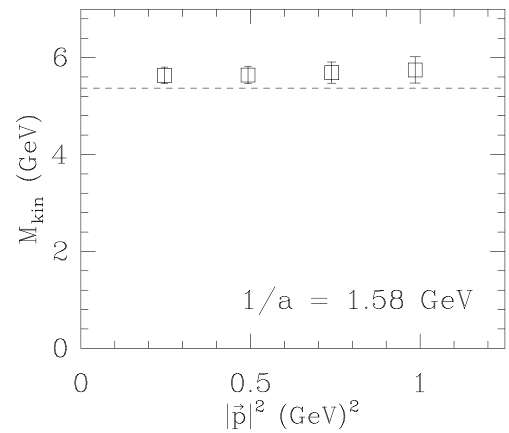

Figure 3. Kinetic B_s mass on the unquenched lattice. The dashed line marks the experimental mass.

gives the quenched result

$$f_{B_s} \;=\; 225 \pm 9(\text{stat}) \pm 20(\text{p.t.})\ \text{MeV}. \tag{9}$$

The 20 MeV uncertainty is the estimate of the $\mathcal{O}(\alpha_s^2)$ error in Z_{A_0}.

3. UNQUENCHED RESULTS

Our work has just begun on the large suite of $2+1$ flavor MILC ensembles (see e.g. [4]). So far we have simulated the B_s system on the configurations with $a \approx 0.13$ fm and dynamical bare masses $au_0m_l = 0.01$ and $au_0m_s = 0.05$ (MILC absorbs a factor of u_0 into their definition of m_0 compared to our convention). Our valence mass is $au_0m_0 = 0.05$. We have used the lattice scale determined from the Υ spectrum, $1/a_\Upsilon = 1.58(2)$ GeV [5]. This scale is a bit higher than the one quoted in [4], $1/a_{r_1} = 1.46$ GeV; however the heavy quark potential parameter r_1 is known only through quark models and so is not as rigorous of a quantity for setting the lattice spacing.

In Fig. 3 we show the kinetic B_s mass, e.g. the result for $\mathbf{p} = (0,0,1)$ gives $M_{\rm kin}(B_s) = 5.63(17)$ GeV. We find that the unquenching dramatically improves the $B_s^* - B_s$ splitting, bringing it up to 42.5 ± 3.7 MeV. The value we compute for $2M_{B_s} - M_\Upsilon = 1.361(8)(17)$ GeV where the first error is dominated by the statistical error in $E_{\rm sim}(B_s)$ and the second is due to the uncertainty in a_Υ. Using a_Υ implies that the strange sector actually corresponds to $au_0m_0 \approx 0.04$ [6]. From the physical $M_{B_s} - M_B$ we estimate our value of $2M_{B_s} - M_\Upsilon$ obtained with $au_0m_0 = 0.05$ is too high by about 45 MeV, or $2M_{B_s} - M_\Upsilon = 1.316(8)(17)$ GeV versus 1.278(5) GeV from experiment. Simulations with smaller au_0m_0 will permit a more rigorous interpolation to M_{B_s}.

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